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IMPROVED CHANNEL ALLOCATION FOR MULTICARRIER CDMA WITH ADAPTIVE FREQUENCY HOPPING AND MULTIUSER DETECTION

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Multicarrier code-division multiple access (MC-CDMA) system with adaptive frequency hopping (AFH) has attracted attention of researchers due to its excellent spectral efficiency. A suboptimal water-filling (WF) channel allocation algorithm was previously proposed for the reverse link of this system. To overcome the limitations of the WF algorithm in the presence of fading-induced near-far problem, a new allocation algorithm is proposed and demonstrated to improve performance when the conventional matched filter (MF) receiver is employed. Moreover, the allocation methods are extended to accommodate multiuser detectors (MUDs) at the receiver for MC-CDMA system with AFH. It is demonstrated that the combination of the improved allocation algorithm and the linear MUDs is very efficient in mitigating the fading and multi-access interference (MAI) for realistic mobile radio channels with correlated subcarriers, channel state information (CSI) mismatch, and imperfect power control. Numerical results show that the proposed adaptive transmission method has much greater system capacity than conventional non-adaptive MC direct-sequence (DS)-CDMA system.

Index Terms — Code division multiple access, Multiuser channels, Fading channels, Frequency hop communication, Resource management

I. INTRODUCTION

Multicarrier code-division multiple access (MC-CDMA) methods have been extensively investigated and proposed for future mobile radio systems due to their high spectral efficiency, the interference suppression capability, the exploitation of frequency diversity, and the ability to adapt to time-varying channel conditions [1,2,3,4]. During the past decade, several MC-CDMA systems with adaptive frequency hopping (AFH) were proposed for cellular communication systems in [5,6,7] and have attracted significant attention due to their excellent spectral efficiency. In these systems, the data of each user is multiplexed over one or several substreams, and multiple subcarriers are employed to transmit the substreams of all users. Direct sequence spread spectrum (DS/SS) codes are assigned to all substreams in the system. Each substream is transmitted on one subcarrier. The subcarriers are allocated using the knowledge of the channel state information (CSI). MC-CDMA with AFH exploits both frequency and multiuser diversity and was shown to improve on non-adaptive MC direct-sequence (DS)-CDMA system in [8].

In [6], a MC-CDMA system with AFH was proposed for the forward link. In this system, one substream/user is employed, and each user selects the subcarrier with the largest fading amplitude for transmission. This allocation algorithm provides significant performance improvement over other diversity techniques such as non-adaptive MC DS-CDMA with the maximum ratio combining (MRC) [8]. However, this simple allocation algorithm does not take into account the multiple-access interference (MAI) present in realistic wireless systems. A near-optimal allocation algorithm was utilized in [9] to maximize the total signal to interference and noise ratio (SINR) and was shown to improve on the performance of the simple allocation policy in [6].

For the reverse link, a MC-CDMA system with AFH was investigated in [5]. In this system, multiple substreams are employed for each user, and random signature sequences are assigned for all substreams, resulting in enhanced MAI and intra-user interference. A sub-optimal water-filling (WF) allocation algorithm was proposed, and it was demonstrated this system has better

performance than DS-CDMA system with RAKE receiver and the non-adaptive MC DS-CDMA system [5]. However, the WF algorithm offers limited protection to weaker users. Thus, it suffers from the near-far problem caused by short-term fading and imperfect power control. As a result, the bit error rate (BER) of the WF method is degraded in moderate-to-high signal-to-noise ratio (SNR) region.

In this paper and [10, 17], we investigate an improved MC-CDMA system with AFH for the reverse link. This system exploits realistic quasi-synchronous reverse link model [11] and utilizes orthogonal spreading sequences for all substreams of the same user, thus eliminating intra-user interference. Moreover, we propose a new allocation algorithm to overcome the limitations of the WF algorithm in [5], and demonstrate the resulting BER improvement.

MAI severely limits performance of spread spectrum systems. While it has been demonstrated that multiuser detector (MUD) can greatly improve spectral efficiency of DS-CDMA [12] and non-adaptive MC-CDMA [1,13], to the best of our knowledge, it has not been utilized in MC-CDMA with AFH. In this paper, we employ the MUDs jointly with the proposed allocation algorithm by utilizing the SINR analysis [10,33]. Performance gains achieved using the MUD combined with the proposed allocation method are demonstrated. To reduce the complexity of the allocation algorithm combined with linear MUDs, recursive update of the matrix inverse is utilized.

In previous research on MC-CDMA with AFH [5, 6], perfect knowledge of the CSI at the transmitter and independent fading for all subcarriers were assumed in the performance evaluation. These assumptions are too optimistic for realistic wireless channels. The impact of noisy CSI measurements was considered in [6, 7]. However, imperfect CSI is mainly caused by the feedback delay associated with sending channel allocation instructions from the base station to mobile users in a rapidly varying fading channel. While the impact of imperfect CSI has been studied extensively for other adaptive transmission systems [14, 15, 22, 23], this investigation has not been carried out for the MC-CDMA system with AFH. In addition, it is well known that

correlated subcarriers reduce the frequency diversity of MC systems [16], and the assumption of independent subcarriers tends to overestimate the system performance [17]. In this paper, both imperfect CSI and correlated subcarriers are considered in the evaluation of system performance.

The remainder of this paper is organized as follows. In section II, we describe the reverse link model of the proposed MC-CDMA system with AFH and MUDs employed in this paper. Section III analyzes the limitations of the WF allocation method, and describes the proposed allocation algorithm. Numerical results and conclusion are contained in sections IV and V, respectively. Symbols frequently used in this paper are summarized in Table I.

II. SYSTEM MODEL

A. Transmitted Signal

System diagram of MC-CDMA with AFH is illustrated in Fig. 1. In the system with K users, the total bandwidth W is divided into M subcarriers with equal bandwidths $\Delta f = W/M$. The data stream of each user is multiplexed over N substreams, and all substreams are spread by spreading codes in time domain. The low-pass equivalent signal at time t for the n th substream of the k th user is (see Table I)

$$x_{k,n}(t) = \sqrt{2E_b} \sum_{l=-\infty}^{\infty} \sum_{i=0}^{PG-1} b_{k,n}(l) c_{k,n}(i) h(t - lT_b - iT_c) \quad (1)$$

where $b_{k,n}(l) \in \{-1, +1\}$ are the binary phase shift keying (BPSK) modulated information symbols. The spreading code vector for this substream is defined as $\mathbf{c}_{k,n} = [c_{k,n}(0) \ c_{k,n}(1) \ \dots \ c_{k,n}(PG-1)]^T$ with $\|\mathbf{c}_{k,n}\| = 1$, where $\|\cdot\|$ is the Euclidean norm, and PG is the processing gain. The chip waveform $h(t)$ satisfies $\int_{-\infty}^{\infty} h^2(t) dt = 1$.

After spreading, the substream (k, n) is assigned to the $q_{k,n}$ th subcarrier, where $q_{k,n} \in [1, \dots, M]$. The subcarrier allocation is performed by a control unit at the base station based on the CSI received from the mobiles, and the channel allocation instructions $\{q_{k,n}\}$ are sent to each mobile via a forward control channel. As in [5], more than one substream of the same user can hop onto

the same subcarrier. The total number of substreams assigned to the m th subcarrier is P_m . Note the bit energy E_b is the same for each substream independent of its subcarrier allocation. Thus, power control that adapts to large-scale fading variation is assumed. Better system performance can be achieved by combining subcarrier allocation and per-subcarrier power control, i.e., the power control that compensates for small-scale fading variations. However, the latter would increase the required feedback rate from the base station to the mobile users. Moreover, joint allocation of subcarrier and power is very complex. In contrast, our method exploits both frequency and multiuser diversity while maintaining relatively low complexity.

While it is not practical to maintain orthogonality among substreams of different users [5, 11], it is feasible to require it for each user k , i.e., $\mathbf{c}_{k,n}^T \mathbf{c}_{k,n'} = 0$ for $n' \neq n$. These orthogonal codes are obtained by first assigning random spreading codes to all users and then multiplying the substreams of user k by orthogonal Walsh codewords [10, 18].

The low-pass equivalent transmitted signal for the k th user is the sum of modulated substream signals $S_k(t) = \sum_{n=1}^N x_{k,n}(t) \exp(-j2\pi f_{q_{k,n}} t)$, where $f_{q_{k,n}}$ is the subcarrier frequency offset from the carrier frequency f_c for the $q_{k,n}$ th subcarrier.

B. Channel Model and Imperfect CSI

We assume slowly varying flat Rayleigh fading for each subcarrier. The channel coefficients $\gamma_{k,m}(t)$ of all subcarriers are identically distributed with $E[|\gamma_{k,m}(t)|^2] = 1$, and are independent for different users. Denote the amplitude $\alpha_{k,m}(t) = |\gamma_{k,m}(t)|$ and the phase $\phi_{k,m}(t) = \angle \gamma_{k,m}(t)$.

To assess realistic system performance, we employ correlated subcarriers. Assuming the propagation delay is exponentially distributed with rms delay spread σ_d [19, p. 50], the cross-correlation between $\gamma_{k,m}(t)$ and $\gamma_{k,m'}(t)$ with index difference $\Delta m = m - m'$ is given by [20]

$$r_f(\Delta m \Delta f) \triangleq \frac{E[\gamma_{k,m}(t) \gamma_{k,m'}^*(t)]}{\sqrt{E[|\gamma_{k,m}(t)|^2] E[|\gamma_{k,m'}(t)|^2]}} = \frac{1 + 2j\pi \Delta m \Delta f \sigma_d}{1 + (2\pi \Delta m \Delta f \sigma_d)^2} \quad (2)$$

In the reverse link, the base station accurately estimates the current CSI of all users at each subcarrier by employing pilot signals [21]. However, due to the delay caused by the calculation of allocation variables and feeding the allocation assignments back to mobile users, the CSI used for allocation is different from the actual channel conditions experienced during the transmission. An efficient approach to reliable transmission in the presence of CSI mismatch is to predict future CSI $\gamma_{k,m}(t)$ based on outdated observations using the autoregressive (AR) model-based linear prediction (LP) $\hat{\gamma}_{k,m}(t)$ [14, 22]. Since the random variables $\hat{\gamma}_{k,m}(t)$ and $\gamma_{k,m}(t)$ are jointly Gaussian, the reliability of prediction is characterized by the subcarrier-independent cross-correlation $\eta = E[\gamma_{k,m}(t)\hat{\gamma}_{k,m}^*(t)]/\sqrt{E[|\gamma_{k,m}(t)|^2]E[|\hat{\gamma}_{k,m}(t)|^2]}$ [23,24]. The normalized prediction mean square error (NPMSE) is given by $1-\eta^2$ [23]. For example, $\eta=0.925, 0.95, 0.975$ and 1.0 correspond to the NPMSE=0.144, 0.098, 0.049, and 0, i.e., poor, degraded, reliable and perfect prediction, respectively [22, 23]. Implementation of reliable practical prediction algorithm for MC-CDMA with AFH is outside the scope of this paper. To evaluate performance for prediction accuracy specified by the reliability parameter η in simulations, we generate predicted channel coefficients as $\hat{\gamma}_{k,m}(t) = \eta^2 \gamma_{k,m}(t) + \omega(t)$, where $\omega(t)$ is white Gaussian noise with variance $\eta^2(1-\eta^2)$ that is independent of $\gamma_{k,m}(t)$.

C. The Receiver

At the base station, the low-pass equivalent received signal is

$$r(t) = \sum_{k=1}^K \sum_{n=1}^N \alpha_{k,q_{k,n}}(t) x_{k,n}(t - \tau_k) \exp \left\{ j \left[\varphi_{k,n}(t) - 2\pi f_{q_{k,n}} t \right] \right\} + \nu(t) \quad (3)$$

where $\nu(t)$ is the complex additive white Gaussian noise with power spectral density N_0 , τ_k is the timing offset of the k th user, and $\varphi_{k,n}(t) = 2\pi f_{q_{k,n}} \tau_k + \phi_{k,q_{k,n}}(t)$. In this paper, we assume perfect knowledge of the timing and frequency offsets for all users, and perfect CSI at the receiver. As discussed in [11], we adopt the quasi-synchronous assumption that is suitable for the reverse link

in practice, i.e., the timing of all users is aligned within a small synchronization window of one to a few chips in length.

Without loss of generality, we focus on detection of the symbol $b_{k,n}(l)$ at the output of the correlator at the receiver and drop the symbol index l . Denote $\gamma_{k,m} \stackrel{\Delta}{=} \gamma_{k,m}(t)$, $\alpha_{k,m} \stackrel{\Delta}{=} \alpha_{k,m}(t)$, and $\phi_{k,m} \stackrel{\Delta}{=} \phi_{k,m}(t)$ for all k and m during this symbol period.

Define U_m of size P_m as the index set of all substreams allocated to the m th subcarrier, i.e. $U_m = \{(k, n) \mid q_{k,n} = m\}$. Without loss of generality, we replace the substream index pair $(k, n) \in U_m$ with a single index p , and assume the corresponding $\phi_{k,m} = 0$, $\tau_k = 0$, and $\alpha_p \stackrel{\Delta}{=} \alpha_{k,m}$. The output of the correlator for the p th substream is given by [5, 6, 8]

$$Z_p = \tilde{S}_p + \tilde{I}_p + \tilde{N}_p, \quad p \in [1, \dots, P_m] \quad (4)$$

where $\tilde{S}_p = \sqrt{E_b} \alpha_p b_p$ carries the desired bit information, and \tilde{I}_p is the interference term

$$\tilde{I}_p = \sqrt{E_b} \sum_{\substack{p' \in U_m \\ \& p' \neq p}} \alpha_p b_{p'} \rho_{p'p} \quad (5)$$

where $\rho_{p'p}$ is the cross-correlation between the waveforms of corresponding substreams [12]

$$\rho_{(k',n')(k,n)} = \exp[j(\phi_{k',m} - \phi_{k,m})] \int_0^{T_b} \sum_{i_1=0}^{PG-1} [c_{k',n}(i_1)h(t-i_1T_c-\tau_{k'})] \sum_{i_2=0}^{PG-1} [c_{k,n}(i_2)h(t-i_2T_c)] dt \quad (6)$$

In (5), the inter-user interference from adjacent symbols and other subcarriers is negligible [5, 8, 10, 33]. Thus, the interference term in (5) is due primarily to the symbols of other users transmitted on the m th subcarrier that start in the same synchronization window as the desired symbol b_p . Finally, the covariance between the zero-mean Gaussian noise terms \tilde{N}_p in (4) is $E(\tilde{N}_p \tilde{N}_{p'}^*) = N_0 \rho_{p',p}$.

D. SINR Analysis

The received signal (4) is detected and demultiplexed at the receiver to make decisions (see Fig. 1). The analytical SINR of the detector is utilized by the allocation method. The SINR

$\lambda(m,p)$ for the p th substream of the m th subcarrier is given by the ratio of the desired signal power to the total power of residual interference and noise at the input to the decision device.

The SINR of the matched filter (MF) detector is [5, 12]

$$\lambda_{MF}(m,p) = \frac{E_b \alpha_p^2}{\frac{E_b}{PG} \sum_{p'=1}^{P_m} \Delta(p',p) \alpha_{p'}^2 + N_0} \quad (7)$$

where

$$\Delta(p',p) \triangleq \Delta[(k',n'),(k,n)] \triangleq \begin{cases} 1, & \text{if } k' \neq k \\ 0, & \text{otherwise} \end{cases} \quad (8)$$

In (7), we assume perfect synchronization, since this assumption results in the worst case system performance [10,17,18,33].

When the linear decorrelating detector (LDD) is utilized, MAI is completely eliminated in (4), so the SINR is given by the SNR [12, p. 249]

$$\lambda_{LDD}(m,p) = \frac{E_b \alpha_p^2}{N_0 (\mathbf{R}^{-1})_{p,p}} \quad (9)$$

where the $P_m \times P_m$ cross-correlation matrix \mathbf{R} has components $\mathbf{R}_{p,p'} = \rho_{p,p'}$ (see (6)).

The SINR of the linear minimum mean-square error (MMSE) detector can be computed for the p th substream as [12, p. 293-298, 25]

$$\lambda_{MMSE}(m,p) = \left\{ \left[\left(\mathbf{I} + \frac{E_b \mathbf{A} \mathbf{R} \mathbf{A}^H}{N_0} \right)^{-1} \right]_{p,p} \right\}^{-1} - 1 \quad (10)$$

where $\mathbf{A} = \text{diag}\{\alpha_1, \alpha_2, \dots, \alpha_{P_m}\}$, and \mathbf{I} is $k \times k$ identity matrix.

The simple successive interference canceller (SIC) is also employed in this paper. Without loss of generality, assume that $\alpha_1 \geq \alpha_2 \geq \dots \geq \alpha_{P_m}$ [12]. The SINR of the SIC can be approximated as [12, p. 360]

$$\lambda_{SIC}(m,p) \approx \frac{E_b \alpha_p^2}{N_0 + \frac{E_b}{PG} \sum_{p'=p+1}^{P_m} \Delta(p',p) \alpha_{p'}^2 + \frac{4E_b}{PG} \sum_{p'=1}^{p-1} \Delta(p',p) \alpha_{p'}^2 P_{e,SIC}(m,p')} \quad (11)$$

where $P_{e,SIC}(m,p) \approx Q[\sqrt{2\lambda_{SIC}(m,p)}]$ is the BER estimate for the p th substream, $Q(x) = \int_x^\infty \exp(-t^2/2) dt / \sqrt{2\pi}$, and $\Delta(p',p)$ is defined in (8).

III. ALLOCATION ALGORITHMS

In this section, we discuss the limitations of previously proposed WF allocation algorithm and describe a novel allocation method for the reverse link. As discussed in section II (see Fig. 1), these algorithms are implemented at the base station, and the resulting allocation assignments are sent back to the mobile, where they are employed to select the reverse link subcarriers for each substream. Thus, the computational load associated with channel allocation is borne entirely by the base station, which can afford much greater complexity than the mobiles. Moreover, the feedback load is low since only a few allocation bits need to be fed back at a low rate on the order of the maximum Doppler shift.

All allocation methods employ the SINR minimization criterion and utilize the SINR expression $\lambda(m,p)$ in (7), (9), (10), or (11) that depends on the type of the receiver utilized at the base station. Otherwise, the allocation methods do not depend on the receiver design and can be easily adapted to various receiver choices.

Subcarrier allocation methods were also addressed in [26, 27, 28, 29, 30]. However, these papers focus on the orthogonal frequency-division multiple access (OFDMA) system, and the throughput and fairness among users are primary concerns, since the target application is the internet protocol (IP)-based packet transmission system. In contrast, we consider MC-CDMA with AFH that has fixed data rate for all users in the system, which is suitable for such applications as voice and circuit-switched data transmission. Thus, it is reasonable to use the minimum SINR as performance measure to guarantee the quality of service for all users. Moreover, since the multi-access interference (MAI) is not significant in OFDMA, design of the allocation algorithm is simpler for this system than for MC-CDMA.

A. The Waterfilling (WF) Algorithm

The WF algorithm was proposed in [5] for the MF receiver. Note that this method is not the water-filling solution that achieves the information-theoretic capacity, although its design was inspired by the latter. The WF algorithm is an iterative allocation algorithm, i.e., one substream is allocated in each iteration. The sequence of KN iterations is divided into N consecutive groups, with K iterations per group. In each group, one substream per user is allocated. During each iteration of the n th group, all users whose substreams have not yet been allocated in this group select the subcarrier with the highest SINR, and the user that has the lowest selected SINR level is assigned to that subcarrier [5]. The protection of weak users offered by the order of subcarrier allocation in the WF method is limited due to two reasons. First, weak users already assigned to given subcarrier cannot contribute large interference level that would prevent allocation of a much stronger user's substream to this subcarrier. Second, the WF algorithm does not take into account the MAI impact of assigning a new substream on the substreams already allocated to certain subcarrier. As a result, the BER of the WF allocation algorithm is very high for some realizations of channel fading coefficients. This case is demonstrated in the following example.

Example Consider a system with $K=N=M=2$, $PG=16$, $E_b=1$, $N_0=0.01$ and $\eta=1.0$. Suppose fading amplitudes are represented by $\{\alpha_{1,1}^2=0.1, \alpha_{1,2}^2=0.09, \alpha_{2,1}^2=1, \alpha_{2,2}^2=0.1\}$.

The WF algorithm [5] performs the allocation in four iterations. The resulting allocation is $\{q_{1,1}=1, q_{1,2}=2, q_{2,1}=1, q_{2,2}=1\}$. Suppose that substream $(k=1, n=1)$ corresponds to substream $p=1$ of the first subcarrier. From (7), the BER of this substream is approximated as

$$P_{e,MF}(m=q_{1,1}=1, p=1) \approx Q\left(\sqrt{\frac{2 \times 0.1}{0.01 + 2/16}}\right) \approx 0.11.$$

A better allocation solution is to assign the substreams of the first user to the first subcarrier and the substreams of the second user to the second subcarrier, i. e., $\{q_{1,1}=1, q_{1,2}=1, q_{2,1}=2, q_{2,2}=2\}$. The resulting BERs are $P_{e,MF}(m, p) \approx Q(\sqrt{2 \times 0.1 / 0.01}) \approx 3.8 \times 10^{-6}$ for $m, p \in \{1, 2\}$.

This example illustrates that the WF algorithm has poor performance when the instantaneous fading amplitude of one user (user 2 in this example) is much larger than that of the other user. While such events are unlikely, they dominate the average BER in moderate to high SNR region. Therefore, it is desirable to design an allocation algorithm that overcomes the limitations of the WF method.

B. Proposed Allocation Algorithm

Our allocation method has the following optimization criterion. As in [5], we focus on the performance of the substream with the lowest SINR since the error events associated with this substream dominate the error rate. The optimization objective is to maximize the SINR of the allocated substream with the lowest SINR, i.e., to find the set of allocation variables that

$$Q_o = \arg \max_Q \left\{ \min_{\substack{m \in [1, \dots, M] \\ p \in [1, \dots, P_m]}} \lambda(m, p) \right\} \quad (12)$$

where $Q = \{q_{k,n} | k \in [1, \dots, K], n \in [1, \dots, N]\}$ is the set of all possible allocation variables, and $\lambda(m, p)$ is the SINR calculated from (7), (9), (10), or (11), depending on the type of the detector employed at the receiver. In (12), SINRs of all substreams are used for allocation, so the weak user is protected well. Thus, better performance is expected relative to the WF algorithm.

Theoretically, the search over the elements of Q produces the optimal solution for (12). There are $(N+M-1)! / [(M-1)!N!]^K$ possible ways to allocate N equivalent substreams/user into M distinct subcarriers [33]. Thus, the computational complexity of finding the exact solution to (12) is very high, and a suboptimal method with moderate complexity is desirable.

Inspired by the iterative algorithms in [5,9,26,27], we design an iterative allocation method to find a suboptimal solution for (12). In this algorithm, one substream is assigned at each iteration, and all substreams are allocated after KN iterations. For simplicity, the substreams are assigned consecutively, i.e. the substream (k, n) is allocated at the $[K(n-1)+k]$ th iteration. It is shown in [33] that this simple assignment order results in negligible performance degradation relative to more sophisticated methods. The algorithm is based on the following idea. Suppose the

substream (k, n) is allocated to the subcarrier m . We observe that in this case only the substream (k, n) and the substreams previously assigned to this subcarrier experience the SINR degradation. Denote the set of such substreams U'_m . For each m in $[1, \dots, M]$, we determine the lowest SINR level in U'_m assuming that the substream (k, n) is allocated to subcarrier m . Then find the subcarrier m_o that produces the maximum value of this lowest SINR among all M values and allocate the substream (k, n) to subcarrier m_o . In our allocation process, the substreams are allocated consecutively, and the interference from a subset of all substreams is taken into account for each allocation. Therefore, it results in a suboptimal solution to (12). However, it improves upon the WF method in [5] since it protects previously allocated weaker users. The proposed improved allocation algorithm consists of the following steps:

S0) Initialize $P_m=0$ and $U_m=\{\emptyset\}$ for all m in $[1, \dots, M]$

for $n=1$ to N

for $k=1$ to K

S1) Augment $U'_m=\{U_m, (k, n)\}$ for $m \in [1, \dots, M]$. Let $\lambda(m, p)$ be the SINR of the p th substream ($p \in [1, \dots, P_m+1]$) assuming that substream (k, n) is assigned to the m th subcarrier.

S2) Find m_o that satisfies

$$m_o = \arg \max_{m \in [1, \dots, M]} \left\{ \min_{p \in [1, \dots, P_m+1]} \lambda(m, p) \right\}$$

S3) Assign substream (k, n) to the m_o th subcarrier, i.e. set $q_{k, n}=m_o$. Then update

$P_{m_o}=P_{m_o}+1$, and $U_{m_o}=U'_{m_o}$.

When perfect CSI is available at the base station, the SINR in step S1 of the improved allocation algorithm is calculated using the expression (7), (9), (10), or (11), depending on the type of the detector employed at the receiver. As discussed earlier, this perfect knowledge of CSI assumed in the calculation of $\lambda(m, p)$ is usually unavailable. Since an algorithm that takes the prediction error into account is very complex, we focus on a simpler approach that employs the predicted fading amplitude $\hat{\alpha}_p$ instead of α_p in the SINR $\lambda(m, p)$ and obtains the SINR estimate $\hat{\lambda}(m, p)$. Thus, the predicted CSI is treated as if it was perfect, and $\hat{\lambda}(m, p)$ is used as performance

measure in step S1 of our allocation method. The sensitivity of allocation algorithm to the prediction errors is tested using simulations in section V.

C. Complexity Reduction

In our allocation algorithm, the computational complexity is mainly determined by the calculation of the SINR. For MF and SIC receivers, the calculation of the SINR given by (7) and (11), respectively, involves only simple scalar operations. On the other hand, the allocation algorithms for the LDD or MMSE detector are complex due to the matrix inversion in (9) and (10), which has to be performed for every U'_m in step S1 of the improved allocation algorithm. However, it is possible to reduce complexity by recursive computation of this inverse as discussed below for the case of the LDD at the receiver. In this recursion, for each m in step S1, denote the $P_m \times P_m$ cross-correlation matrix of the substreams previously assigned to subcarrier m as $\mathbf{R}(P_m)$. The inverse of this matrix $[\mathbf{R}(P_m)]^{-1}$ has been computed during an earlier iteration. In the current iteration, we need to evaluate the SNR $\lambda_{LDD}(m, p)$, $p \in [1, \dots, P_m + 1]$ when assignment of new $(P_m + 1)$ st substream is examined for subcarrier m in step S1. This computation requires the inversion of an augmented correlation matrix

$$\mathbf{R}(P_m + 1) = \begin{bmatrix} \mathbf{R}(P_m) & \boldsymbol{\rho} \\ \boldsymbol{\rho}^H & 1 \end{bmatrix}, \quad (13)$$

where $\boldsymbol{\rho} = [\rho_{P_m+1,1}, \rho_{P_m+1,2}, \dots, \rho_{P_m+1,P_m}]^T$ is the vector of the cross-correlation coefficients between the new substream and P_m existing substreams determined by (6). The inverse of (13) can be computed by exploiting the existing $[\mathbf{R}(P_m)]^{-1}$ as follows [31, eq. (35)]:

$$[\mathbf{R}(P_m + 1)]^{-1} = \begin{bmatrix} [\mathbf{R}(P_m)]^{-1} + \mu [\mathbf{R}(P_m)]^{-1} \boldsymbol{\rho} \boldsymbol{\rho}^H [\mathbf{R}(P_m)]^{-1} & -\mu [\mathbf{R}(P_m)]^{-1} \boldsymbol{\rho} \\ -\mu \boldsymbol{\rho}^H [\mathbf{R}(P_m)]^{-1} & \mu \end{bmatrix} \quad (14)$$

where $\mu = [1 - \boldsymbol{\rho}^H [\mathbf{R}(P_m)]^{-1} \boldsymbol{\rho}]^{-1}$. This recursion requires about $1.5(P_m + 1)^2$ complex multiplications and divisions [33], which is significantly less than the $(P_m + 1)^{\log_2 7}$ operations of efficient Strassen's method for direct matrix inversion [32]. The SINR calculation for MMSE detector (10) can be simplified similarly [33].

IV. NUMERICAL RESULTS

First, we compare the average BER (over all users and fading realizations) of the WF algorithm, the optimal allocation method (12) and the proposed algorithm for the MF detector in Fig. 2. As in [5], the assumptions of perfect CSI knowledge and independent fading subcarriers are adopted in this simulation, as well as in Fig. 3 and 4. Due to very high complexity of the optimal allocation, small number of substreams is employed in this comparison. We observe that the proposed algorithm incurs a small penalty relative to the optimal allocation method for high SNR and outperforms the latter for low SNR since the average BER is not the optimality criterion. As expected, the WF algorithm has the worst performance among the three allocation algorithms in moderate to high SNR. Fig. 2 also demonstrates that the AFH system with the proposed allocation algorithm provides 2-3dB power gain over the non-adaptive MC DS-CDMA with MRC [8] in this multiuser scenario.

Fig. 3 compares the average BER of the WF method and the proposed allocation algorithm for the MF receiver and a larger number of users in the system. We observe that the performance gain of the proposed algorithm over the WF method increases as the SNR grows. We have also investigated the modification of the WF algorithm when the LDD is employed in the receiver in [33]. Significant loss was also observed relative to the proposed allocation method in this case. The comparison in Fig. 3 corroborates our discussion on the limitations of the WF algorithm in section III-A.

Fig. 4 shows the average BER of the proposed allocation algorithm for moderate system load (K comparable to M) combined with receivers discussed in section II. To illustrate the potential of combined subcarrier allocation and MUD, the single user bound (SUB), i.e., the BER of the system with only one user, is included in the figure. The linear methods (the MMSE detector and LDD) have much better performance than the SIC and the MF detectors for moderate to high SNR. The linear detectors approach the SUB in this system because the adaptive allocation algorithm separates the substreams with high cross-correlations into different subcarriers. This

adaptive allocation policy also reduces the effect of noise enhancement that degrades LDD in conventional CDMA channels, so it is only slightly outperformed by the MMSE detector in our system. We also note that when the allocation method is employed with the SIC, it is impaired in the first few stages by the inaccuracy of the SINR approximation (11) when the number of substreams is small [12].

We investigate the effect of imperfect CSI in Fig. 5. Observe that while the BER of the adaptive system with LDD increases as the reliability of CSI degrades, it is still smaller than the SUB of non-adaptive MC DS-CDMA even for $\eta=0.95$, which is easily achievable in realistic scenarios [15, 22].

Fig. 6 illustrates the impact of correlated subcarriers on the BER of the adaptive system when perfect knowledge of the CSI is assumed. As expected, the performance degrades as $\Delta f\sigma_d$ decreases. However, for $\Delta f\sigma_d$ as small as 0.3, the performance is very close to the case of independent fading subcarriers ($\Delta f\sigma_d=\infty$). Observe that the BER gap between the LDD and SIC narrows as $\Delta f\sigma_d$ decreases in Fig. 6 since fading, not MAI, becomes the dominant source of performance degradation as diversity reduces for small $\Delta f\sigma_d$. Similarly, it is shown in [33] that performance gap between these two detectors narrows when the predicted CSI is unreliable in the adaptive system. The complexity comparison in [33] reveals that the allocation algorithms for SIC and linear detectors have similar complexity. Therefore, the LDD or MMSE detectors provide the best complexity/performance/robustness trade-off in this adaptive system.

In Fig. 7 and 8, we investigate a practical system design with the available bandwidth set to 1.25MHz, the bandwidth of single carrier in IS-95 and CDMA2000 standard. The rms delay spread of the channel is assumed $\sigma_d=1\mu s$, which is typical for the urban environment [18, p. 200]. Since the coherence bandwidth is approximately $1/5\sigma_d=200\text{KHz}$ [18, p. 202], we divide the available bandwidth into 12 subcarriers with the subcarrier bandwidth of 102.4KHz to assure flat fading for each subcarrier. Assuming $N=8$ substreams and $PG=64$, the system supports a 12.8Kbps data service to each user. Moreover, for comparison, we have simulated a non-

adaptive MC DS-CDMA system with the same system bandwidth, number of subcarriers, and data rate as for the adaptive system. To maintain the data rate, the time-domain processing gain of eight is required in the latter system. While in the original MC DS-CDMA design, the same spreading code was used for all subcarriers of one user [8], we employ different random spreading codes for different subcarriers as proposed in [13] to reduce the variance of the cross-correlation between the waveforms of different users.

Fig. 7 shows the BER of the adaptive system for varying numbers of users. For all detectors, the BER level is maintained provided the number of users K is sufficiently small. This is due to the fact that the fading induced near-far problem can be easily suppressed for a small number of users. While the BER of the MF receiver degrades for $K \geq M$, the MUDs maintain reliable performance unless the system load is very large, i. e., when $K \gg M$. The MMSE detector has the best performance, but its gain over the LDD is very small as discussed above.

In Fig. 8, the BERs of the adaptive and non-adaptive systems with the LDD are compared. For the non-adaptive system, this MUD is derived from the RAKE LDD designed for the multipath fading channel in [34]. Even with poorly predicted CSI ($\eta=0.925$), the AFH system outperforms the non-adaptive system. For target BER of 10^{-3} , the adaptive system can support from 12 to 63 users, depending on the quality of the predicted CSI, while only two users are feasible for the non-adaptive system.

In previous simulations, the power control that adapts perfectly to large-scale fading is assumed, i.e., E_b is the same for all users. To investigate the effect of the near-far problem caused by imperfect power control on the proposed allocation algorithm, the SNR of randomly chosen desired user is set to 10dB, while $K-1$ interfering users have SNR=20dB. Given the same system parameters as in Fig. 7, the simulated BER of the desired user is shown in Fig. 9. Note that MUDs in this figure have lower BER than for the perfect power control case in Fig. 7, and the BERs of linear detectors are very close to the system SUB even for heavily loaded near-far scenario. The reason is that the weakest user's substreams are assigned into their favorite

subcarriers by our allocation algorithm, while the interfering stronger users are competing for remaining subcarriers. In previous figures, our allocation policy was shown to alleviate small-scale fading-induced near-far problem. Fig. 9 demonstrates that combined proposed allocation algorithm and MUD are also very effective in compensating for large-scale fading and imperfect power control.

V. CONCLUSIONS

A novel allocation algorithm that outperforms the WF algorithm for MC-CDMA system with AFH for the reverse link was proposed. The proposed allocation algorithm is combined with MUDs using the SINR analysis. It is demonstrated that combined improved subcarrier allocation and MUD is very effective in mitigating MAI, resulting in much larger system capacity than for the non-adaptive MC DS-CDMA system. This conclusion is confirmed for realistic rapidly varying fading channels with correlated subcarriers, CSI mismatch, and imperfect power control.

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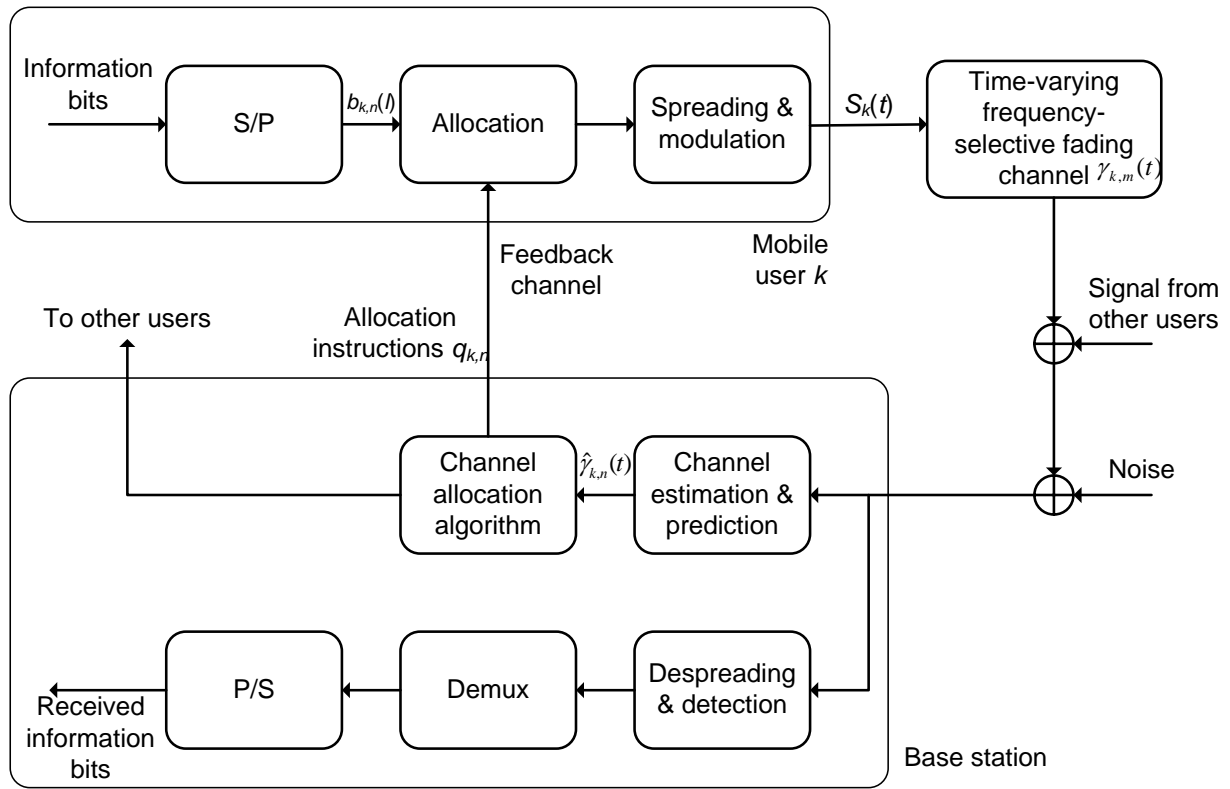


Fig. 1 System diagram of MC-CDMA with AFH for reverse link

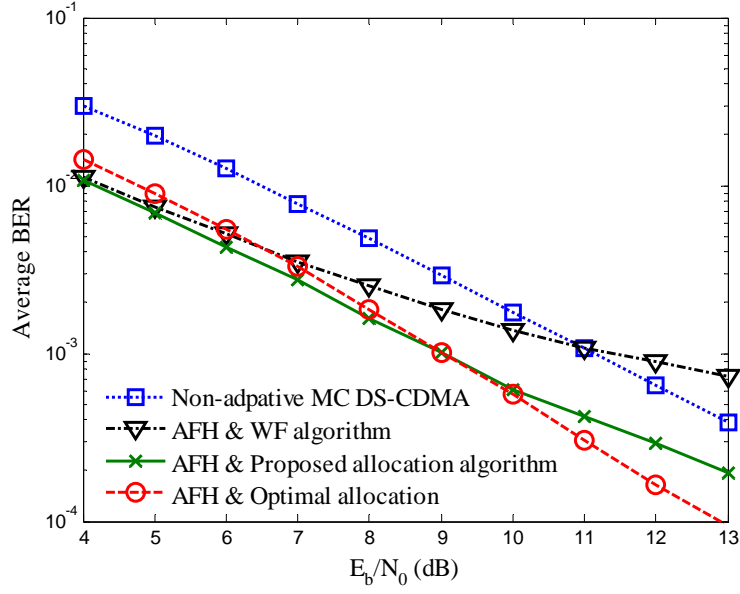


Fig. 2 BER performance of adaptive MC-CDMA with WF, optimal and proposed allocation algorithms ($N=1$ substream/user, $PG=64$ and perfect CSI knowledge), and non-adaptive MC DS-CDMA with MRC (time domain processing gain $PG=64$). For both systems, $K=6$ users, $M=4$ subcarriers, independent fading subcarriers ($\Delta f\sigma_d=\infty$), and MF receiver are assumed.

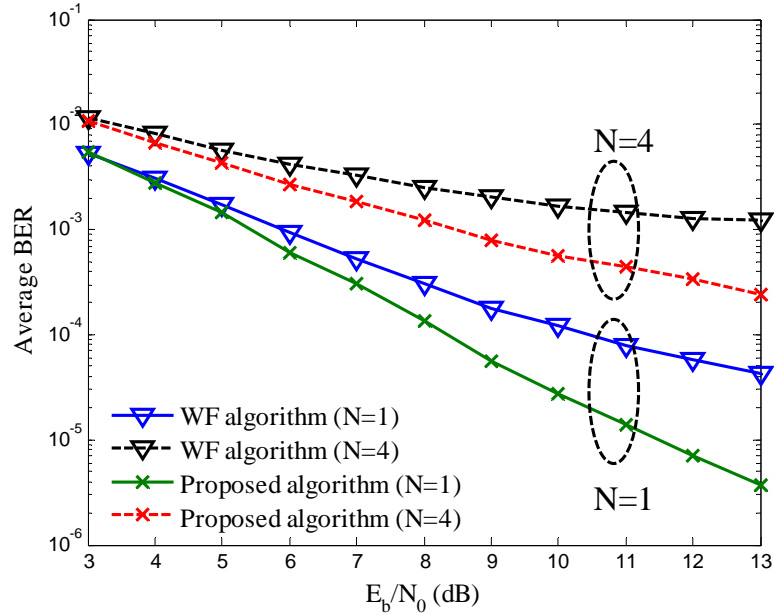


Fig. 3 BER performance of adaptive MC-CDMA with MF receiver for $K=16$, $M=8$, $PG=64$,

$$\Delta f\sigma_d=\infty, \text{ and } \eta=1.0$$

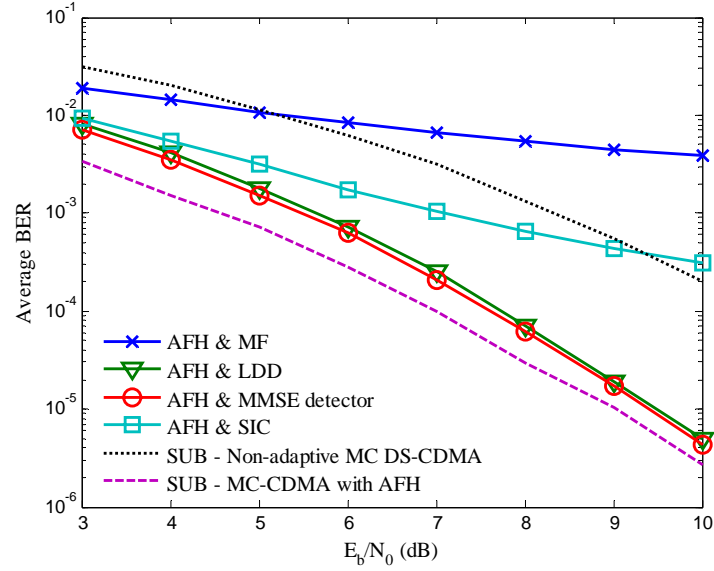


Fig. 4 BER performance of MC-CDMA with AFH for $K=16$, $N=8$, and $\eta=1.0$. In all systems,

$$M=8, PG=64, \text{ and } \Delta f\sigma_d=\infty.$$

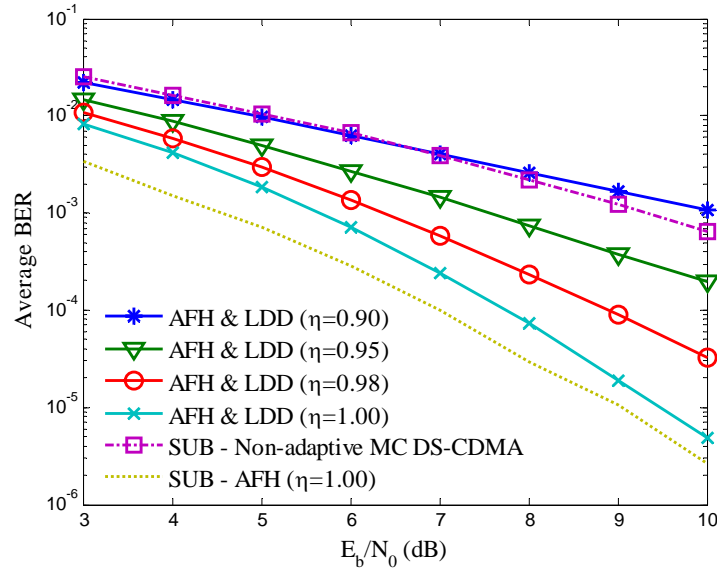


Fig. 5 Comparison of SUB of non-adaptive MC DS-CDMA with the BER of MC-CDMA with AFH and LDD for $K=16$, $N=8$, and varying CSI reliability η . In all systems, $PG=64$, $M=8$, and

$$\Delta f\sigma_d=\infty.$$

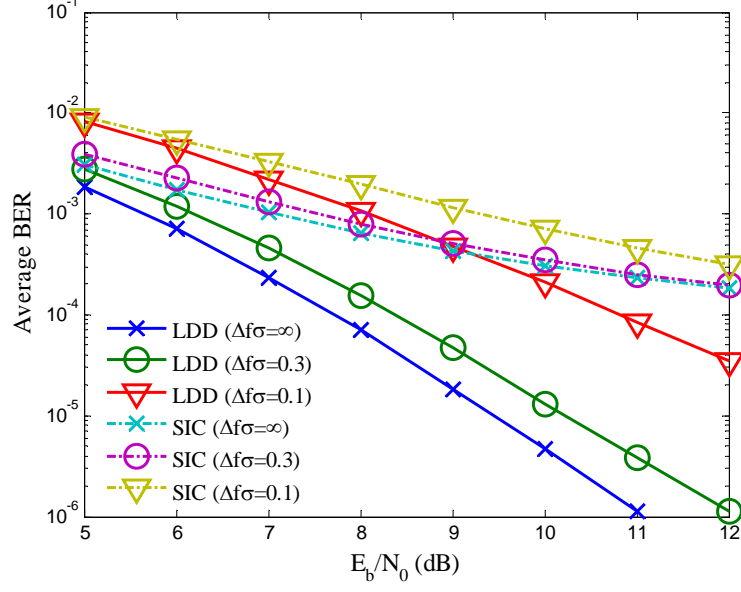


Fig. 6 BER performance of MC-CDMA with AFH and LDD/SIC for $K=16$, $M=8$, $N=8$, $PG=64$, and perfect CSI ($\eta=1.0$) at the allocation algorithm.

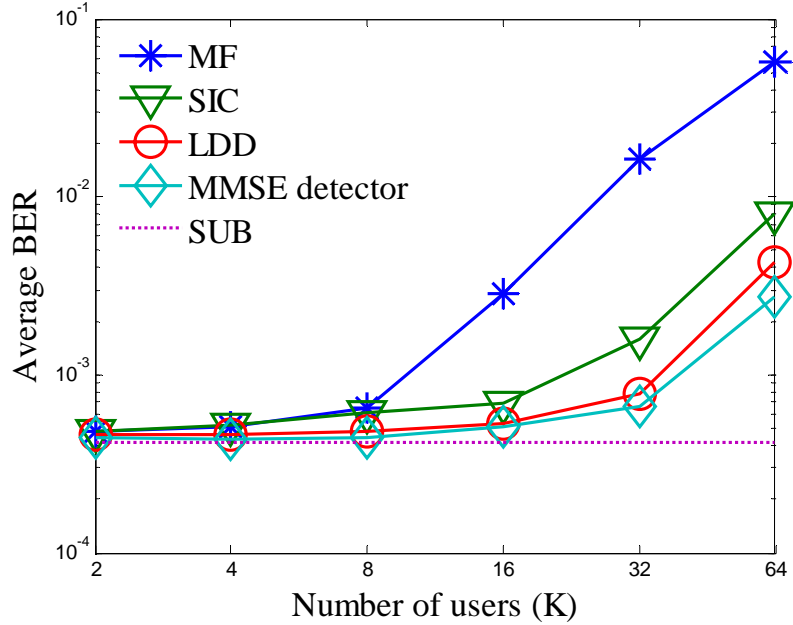


Fig. 7 BER vs. number of users in the adaptive system with improved allocation algorithm for $SNR=10dB$, $M=12$, $N=8$, $PG=64$, $\eta=0.95$, and $\Delta f\sigma_d=0.1024$

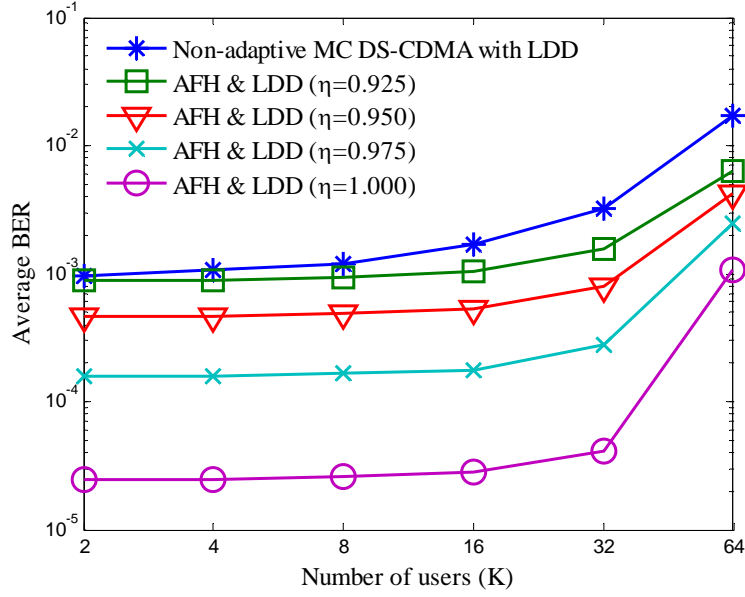


Fig. 8 BER vs. number of users in the system with LDD for SNR=10dB, $M=12$, $N=8$, $PG=64$, and $\Delta f\sigma_d=0.1024$.

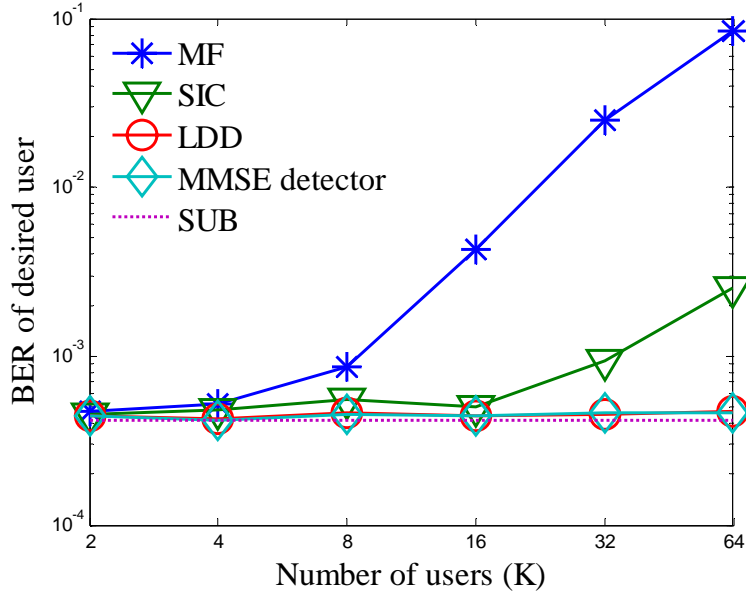


Fig. 9 BER vs. number of users in the adaptive system with imperfect power control for $M=12$, $N=8$, $PG=64$, $\eta=0.95$, and $\Delta f\sigma_d=0.1024$. The SNR is 10dB for desired user and 20dB for other users.

Table I Notation

K	number of users
M	number of subcarriers
N	number of substreams/user
k, k'	user indices
m, m'	subcarrier indices
n, n'	substream indices for a given user
p, p'	substreams indices for a given subcarrier
(k, n)	the n th substream of the k th user
τ_k	timing offset of the k th user
E_b	bit energy
N_0	power spectral density of channel Gaussian noise
T_c	chip duration
T_b	bit duration
PG	processing gain= T_b/T_c
Δf	subcarrier bandwidth
σ_d	rms delay spread
$q_{k,n}$	allocation variable for substream (k, n)
$\gamma_{k,m}(t)$	fading coefficient on the m th subcarrier of user k
$\hat{\gamma}_{k,m}(t)$	prediction of $\gamma_{k,m}(t)$
$\alpha_{k,m}(t), \hat{\alpha}_{k,m}(t)$	the amplitudes of $\gamma_{k,m}(t)$ and $\hat{\gamma}_{k,m}(t)$
$\phi_{k,m}(t)$	phase of $\gamma_{k,m}(t)$
η	cross-correlation between $\gamma_{k,m}(t)$ and $\hat{\gamma}_{k,m}(t)$
U_m	set of all substreams assigned to subcarrier m
P_m	number of substreams in subcarrier m
$\lambda(m, p)$	SINR for the p th substream on subcarrier m